Physical Information and Fundamental Energy Limits in Computation

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The Landauer Limit

Working Definition

Erasure of an amount I_{er} of information from a physical system unavoidably results in an

- entropy increase of

$$\Delta S \geq k_B \ln(2) I_{er}$$

- energy dissipation of

$$\Delta E \geq k_B T \ln(2) I_{er}$$

where

- k_B is Boltzmann's constant.
- T is the environment temperature.
- I_{er} is amount of information lost *irreversibly*.

—Many variations on the theme of R. Landauer, *IBM J. Res. Dev.* **5**, 183 (1961).

Interpretation of Key Quantities

- Entropy & energy of what? Defined and quantified how?
- Information about/of what? Defined and quantified how?
- What, *physically*, counts as info erasure? Irreversible info loss?

Interpretation/Perception of Claim

- Implication of achievability, or involuible bound?
- Just a consequence of the Second Law, or something else?
- Too model dependent? Too model independent?

Perception of Status

- Is Landauer's Limit "extremely well established," or do...
- "we still await a cogent justification of Landauer's Principle"
  —J. Norton, *Stud. Hist. Philosophy Mod. Phys.* **42**, 184 (2011).

Methodological Objections

Motivating Question

How much can be established about the energy cost of irreversible information loss in physical computing contexts...

- as clearly, transparently, rigorously, and generally as possible...
- from as little as possible beyond physical law...
- while addressing or sidestepping common objections?

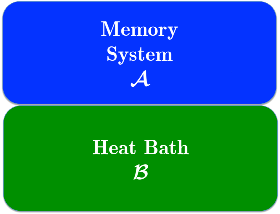
Answers

Sketch proofs of three quantum-dynamical bounds

- Baseline Bound
- Trial-Averaged Bound
- Physical-Informatic Bound (briefly)

Baseline Proof
Setup

Setting



Globally closed composite

- one-bit memory $\mathcal{A}$
- heat bath $\mathcal{B}$

Bath assumed finite and *initially* at temperature T.

Encoding

$$\hat{\rho}^{\mathcal{A}} = \begin{cases} \hat{\rho}_0^{\mathcal{A}} & \text{for “binary 0"} \\ \hat{\rho}_1^{\mathcal{A}} & \text{for “binary 1"} \end{cases}$$

$\hat{\rho}_0^{\mathcal{A}}$, $\hat{\rho}_1^{\mathcal{A}}$ distinguishable and equiprobable

Initial State

$$\hat{\rho}^{\mathcal{A}} = \tfrac{1}{2}\hat{\rho}_0^{\mathcal{A}} + \tfrac{1}{2}\hat{\rho}_1^{\mathcal{A}}$$
$$\hat{\rho}_{th}^{\mathcal{B}} = \exp[-\hat{H}^{\mathcal{B}}/k_B T]$$

Final State (“Reset-to-Zero" erasure)

$$\hat{\rho}^{\mathcal{AB}'} = \hat{U}\left(\hat{\rho}^{\mathcal{A}} \otimes \hat{\rho}_{th}^{\mathcal{B}}\right)\hat{U}^{\dagger}$$
$$\hat{\rho}^{\mathcal{A}'} = Tr_{\mathcal{B}}[\hat{\rho}^{\mathcal{AB}'}] = \hat{\rho}_0^{\mathcal{A}}$$
$$\hat{\rho}^{\mathcal{B}'} = Tr_{\mathcal{A}}[\hat{\rho}^{\mathcal{AB}'}]$$

Change in Bath Energy

$$\boxed{\Delta\langle E^{\mathcal{B}}\rangle = \langle E^{\mathcal{B}'}\rangle - \langle E^{\mathcal{B}}_{th}\rangle} = Tr[\hat{\rho}^{\mathcal{B}'}\hat{H}^{\mathcal{B}}] - Tr[\hat{\rho}^{\mathcal{B}}_{th}\hat{H}^{\mathcal{B}}]$$

Lower Bound on $\Delta\langle E^{\mathcal{B}}\rangle$: Proof Ingredients

- Partovi's Inequality: $\Delta\langle E^{\mathcal{B}}\rangle \geq k_B T \ln(2)\Delta S^{\mathcal{B}}$.
  - $S^{\mathcal{B}}$: von Neumann entropy of bath $\mathcal{B}$
  - T: *initial* temperature of bath $\mathcal{B}$
- Subadditivity of von Neumann entropy
- Invariance of von Neumann entropy under unitary evolution

Result: Bound on $\Delta\langle E^{\mathcal{B}}\rangle$

$$\langle E^{\mathcal{B}}\rangle \geq k_B T \ln(2)$$

Features

- Bound follows exclusively from dynamical law and entropic inequalities.
- No equilibrium or quasi-static assumptions: T refers only to initial temperature of finite bath

Limitations

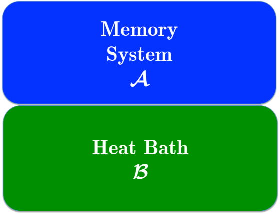
- Very Special Case
  - Symmetric, one-bit memory
  - Uniform encoding probabilities
- What about information? Reversibility?

Objection

- Unsubstantiated use of “average" initial state
  - Initial state is only ever $\hat{\rho}_0^{\mathcal{A}}$ **OR** $\hat{\rho}_1^{\mathcal{A}}$, but proof is for $\hat{\rho}^{\mathcal{A}} = \frac{1}{2}\hat{\rho}_0^{\mathcal{A}} + \frac{1}{2}\hat{\rho}_1^{\mathcal{A}}$

Trial-Averaging Proof
Setup

Setting



Globally closed composite

- memory system $\mathcal{A}$
- heat bath $\mathcal{B}$

Bath assumed finite and *initially* at temperature T.

Encoding

$$\hat{\rho}^{\mathcal{A}} = \hat{\rho}_i^{\mathcal{A}} \text{ for symbol } x_i$$

$\hat{\rho}_i^{\mathcal{A}}$ are mutually distinguishable

x_i is encoded with probability p_i

Initial State (trial w/ x_i encoded)

$$\hat{\rho}^{\mathcal{A}} = \hat{\rho}_i^{\mathcal{A}}$$
$$\hat{\rho}_{th}^{\mathcal{B}} = \exp[-\hat{H}^{\mathcal{B}}/k_B T]$$

Final State (reset $\hat{\rho}_i^{\mathcal{A}}$ to $\hat{\rho}_{reset}^{\mathcal{A}}$)

$$\hat{\rho}_i^{\mathcal{AB}'} = \hat{U}_i \left(\hat{\rho}_i^{\mathcal{A}} \otimes \hat{\rho}_{th}^{\mathcal{B}}\right) \hat{U}_i^{\dagger}$$
$$\hat{\rho}_i^{\mathcal{A}'} = Tr_{\mathcal{B}}[\hat{\rho}_i^{\mathcal{AB}'}] = \hat{\rho}_{reset}^{\mathcal{A}}$$
$$\hat{\rho}_i^{\mathcal{B}'} = Tr_{\mathcal{A}}[\hat{\rho}_i^{\mathcal{AB}'}]$$

Change in Bath Energy

$$\boxed{\langle\Delta\langle E^{\mathcal{B}}\rangle\rangle = \sum_i p_i \left(\langle E_i^{\mathcal{B}'}\rangle - \langle E_{th}^{\mathcal{B}}\rangle\right)} = \sum_i p_i \left(Tr[\hat{\rho}_i^{\mathcal{B}'}\hat{H}^{\mathcal{B}}] - Tr[\hat{\rho}_{th}^{\mathcal{B}}\hat{H}^{\mathcal{B}}]\right)$$

Lower Bound(s) on $\langle\Delta\langle E^{\mathcal{B}}\rangle\rangle$: Proof Ingredients (beyond baseline)

- Linearity of unitary-similarity transformations
- Grouping property of von Neumann entropy

Results: Bounds on $\langle\Delta\langle E^{\mathcal{B}}\rangle\rangle$

$\langle\Delta\langle E^{\mathcal{B}}\rangle\rangle \geq k_B T \ln(2)\left[-\langle\Delta S_i^{\mathcal{A}}\rangle\right]$ for conditional reset

$\langle\Delta\langle E^{\mathcal{B}}\rangle\rangle \geq k_B T \ln(2)\left[I_{er}^{\mathcal{A}} - \langle\Delta S_i^{\mathcal{A}}\rangle\right]$ for unconditional reset ($\hat{U}_i = \hat{U}\ \forall\ i$)

- $I_{er}^{\mathcal{A}} = -\sum_i p_i \log_2 p_i = H(X)$: Shannon information erased from $\mathcal{A}$
- $\langle\Delta S_i\rangle = \sum_i p_i \left[S(\hat{\rho}_{reset}^{\mathcal{A}}) - S(\hat{\rho}_i^{\mathcal{A}})\right]$: trial-average entropy change of $\mathcal{A}$

—N.G. Anderson, "Conditional Erasure and the Landauer Limit." In: Lent C., Orlov A., Porod W., Snider G. (eds) *Energy Limits in Computation*. Springer, (2019).

Features

- Use of average initial state sidestepped—*and vindicated*
- Holds for asymmetric memory; nonuniform encoding statistics; general reset state
- Resolves two distinct contributions
  - “information-bearing entropy"—with Shannon entropy of encoding *emerging* as info measure
  - “non-information-bearing entropy"; trial-averaged entropy change
- Connection between conditioning and reversibility explicit

Limitations

- Holds only for distinguishable encoding states
- Not scalable to logic, FSAs, complex computing contexts

Objection

- Physicality and role of information insufficiently clear

We should not *expect* to have
a rigorous, agreed upon
physical quantification of the costs of information processing
in computational contexts
without
a rigorous, agreed upon
physical conception and quantification of information
in computational contexts.

We need a "strongly physical" conception of information (SPCI)
for computational contexts

Candidate SPCI: Observer-local referential (OLR) information

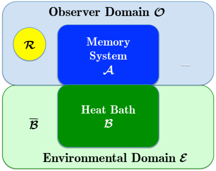
—N.G. Anderson, "Information as a Physical Quantity", *Information Sciences* **415**, 397 (2017).

Physical-Informatic Proof
OLR-Information Measure

OLR Information

$$I^{\mathcal{R}\mathcal{A}} = S(\hat{\rho}^{\mathcal{R}};\hat{\rho}^{\mathcal{A}}) \text{ for } \mathcal{R} \in \mathcal{O}$$
$$I^{\mathcal{R}\mathcal{A}} = 0 \qquad\qquad \text{ for } \mathcal{R} \in \mathcal{E}$$

$S(\circ;\circ)$: correlation entropy (or QMI)
$\hat{\rho}^{\mathcal{R}}$: state of referent system $\mathcal{R}$
$\hat{\rho}^{\mathcal{A}}$: state of info-bearing system $\mathcal{A}$



—N.G. Anderson, "Information as a Physical Quantity", *Information Sciences* **415**, 397 (2017).

Results: Bounds on $\Delta\langle E^{\mathcal{B}}\rangle$

$\Delta\langle E^{\mathcal{B}}\rangle \geq k_B T \ln(2)\left[-\langle \Delta S_i^{\mathcal{A}}\rangle\right]$ conditional reset (general $\hat{U} = \hat{U}^{\mathcal{R}\mathcal{A}\mathcal{B}}$)

$\Delta\langle E^{\mathcal{B}}\rangle \geq k_B T \ln(2)\left[I_{er}^{\mathcal{A}} - \langle \Delta S_i^{\mathcal{A}}\rangle\right]$ unconditional reset ($\hat{U} = \hat{U}^{\mathcal{R}} \otimes \hat{U}^{\mathcal{A}\mathcal{B}}$)

- $I_{er}^{\mathcal{R}\mathcal{A}} = I^{\mathcal{R}\mathcal{A}} - I^{\mathcal{R}\mathcal{A}'} = \chi$: Holevo info of encoding ensemble $\{p_i, \hat{\rho}_i^{\mathcal{A}}\}$

—N.G. Anderson, "Landauer's Limit and the Physicality of Information," *Eur. Phys. J. B* **91**, 156 (2018).

- Bound is identical to that proven by trial-averaging, but is...
  - **generalized** to arbitrary (e.g. noisy) encoding states.
  - **scalable** to logic, FSAs, complex computing contexts
- Based on information measure that...
  - formalizes information as a physical state quantity (of $\mathcal{RA}$)
  - distinguishes states of $\mathcal{A}$ that do and do not bear information
  - harmonizes with conceptions of information in computing contexts
- All relevant copies and records are physically embodied
  - No ghostly “knowers" of information or “conditioners" of operations

  —N.G. Anderson, “Information: The ghost in the computing machine?" forthcoming.
- Bound provable using average initial states *or* trial averaging

The Landauer Limit has a complex history—still controversial!

- Resolution would aid evaluation of reversible computing
- Most objections are to standard thermodynamic approaches

Quantum dynamical approaches can help

- Reveals LL as a transparent consequence of dynamical law
- Enables substantial generalizations
- Addresses or sidesteps key objections
- Clarifies link between conditioning and reversibility
- Solidifies physical meaning(s) and role of “information" in LL
- Enables “scalability" of LL to complex, noisy computing scenarios

Thank You